Chapter 5
Ice-Affected Stage-Frequency Analysis

5-1. Introduction

The purpose of this chapter is to provide techniques for the determination of stage-frequency distributions for rivers subject to periods of ice. Such analyses can be important for projects dealing with ice-affected flooding, clearance for bridges or hydraulic structures, tailwater elevations for hydroelectric plants, water intake elevations, and shore protection design. In contrast to open-water flooding, where high water levels directly result from excessive water discharge, ice-affected flooding results from added resistance to flow and blockage of flow caused by accumulations of ice. Water discharge during ice-induced flooding is typically low relative to open-water floods. Consequently, a flood-frequency analysis based on peak annual discharges will often miss most, if not all, ice-affected events, even though the stages may be among the highest on record. Thus, ice-induced flooding must be analyzed in terms of stage frequency, which is primarily influenced by the ice regime.

5-2. Ice Effects on River Stage and Flooding

The formation of an ice cover or ice jam on a river roughly doubles the wetted perimeter of a wide channel. The added resistance to flow, along with the reduction in flow area caused by the ice, results in higher stages than a comparable open-water discharge would produce. This is particularly true for the case of ice jams, which can cause flood stages comparable to rare open-water events, despite discharge exceedance probabilities on the order of 0.5 or greater. These accumulations include freezeup jams, formed by the collection of pieces of floating ice during the periods of relatively steady flow experienced when the ice cover initially forms early in the winter season, as well as breakup jams, which form during the often highly unsteady flow conditions when the ice cover breaks up because of a significant rainfall event, snow melt, or other increase in runoff.

a. Ice jam flow profiles. Most ice jams are the result of ice moving downstream until it encounters an intact downstream ice cover, or other surface obstruction. Figure 5-1 illustrates the longitudinal profile of a typical fully developed jam. Downstream from the jam, the flow may be uniform (at least in a reach-averaged sense). At the downstream end, or toe, of the jam, the ice accumulation results in a gradually varied flow profile in the transition reach, as water depth increases toward the deeper normal-flow depth associated with the thicker, rougher ice conditions. If the ice jam is long enough, a fully developed or equilibrium-jam reach may form, in which ice and flow conditions are relatively uniform. From the upstream end, or head, of the jam, flow depths again transition toward the lower uniform-flow depth associated with the open-water conditions upstream.

b. Ice jam data. Ice-related flooding tends to be local and highly site specific. While ice jams may be relatively common at a given site, they cannot be predicted with certainty in any given year, and may be totally absent at other sites nearby, even along the same river. Without prior field observations, it is generally difficult to predict where, or even if, jams will form along a river. Thus, ice-affected frequency analysis emphasizes historical data, even though they may be scarcer and less reliable than the open-water data. The available information may range from detailed hydrographic records to observations by local residents. At a minimum, it is necessary to have some information on where, when, and with what frequency ice events happen. The types of available data will determine the form and reliability of the frequency analysis that can be conducted.

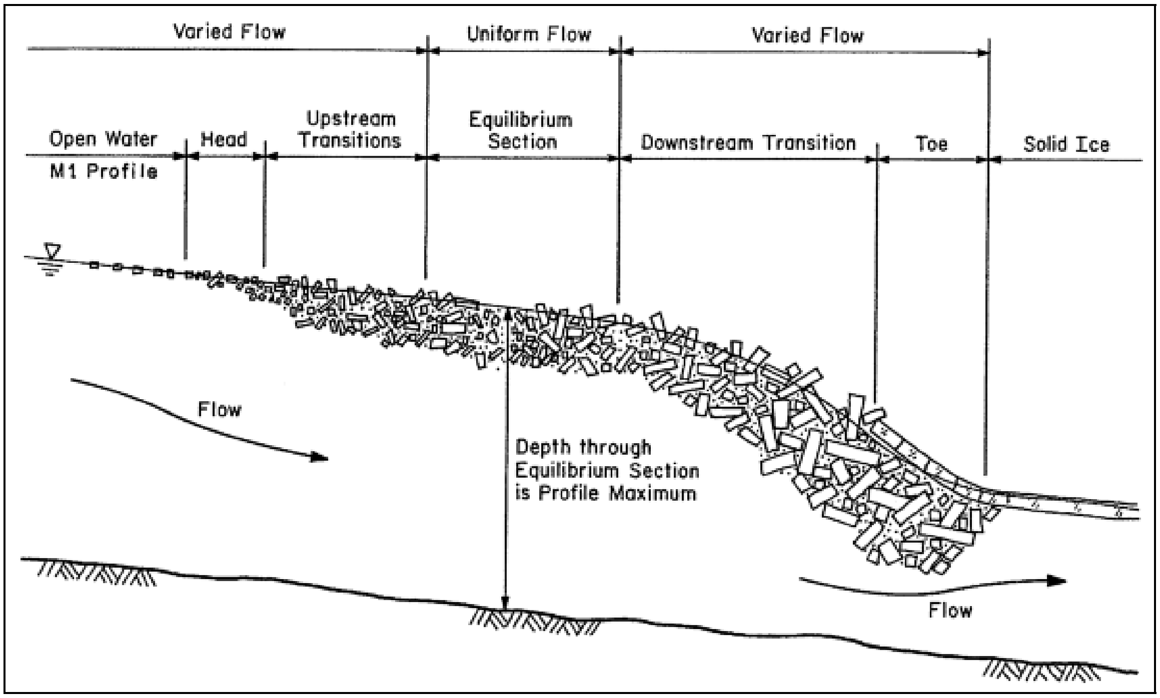

Figure 5-1. Typical ice jam profile

5-3. Data Sources

Ice jam floods often occur when flow rates are relatively low, perhaps no more than a 0.5-exceedance-probability discharge, and water levels are normally high only in the vicinity of the ice and in a backwater zone upstream. Their relatively small geographic extent (perhaps a few river miles or kilometers) and short duration (from a few hours to a few days for breakup events) make it unlikely that detailed field information will have been gathered at most sites. Even in cases where hydrographic gaging records exist for a site, the potential for gage freezeup because of cold weather, ice effects on the gage rating curve, the location of the gage relative to the ice accumulation, or direct ice action on the gage can reduce their reliability for ice events. Because ice jams are site-specific, it is generally not possible to transpose stage data from other sites along the river. Hence, it is often necessary to resort to other sources of data, sources that are often overlooked or regarded as unreliable for analysis of open-water flooding. Sources of historical data might include:

- USGS or Corps gaging station reports or files.
- Corps District or Area Office files.
- State and local water resources and Civil Defense offices.

- Prior flood insurance studies.
- Historical societies, museums, town offices, libraries.
- Newspapers, books, photographs.
- Interviews with local residents.
- Environmental indicators, such as tree scars, structural markings or damage, and vegetation trim lines.

5-4. Form of Frequency Analysis

To analyze ice-influenced flood frequency, mixed populations must be considered. Depending on the objectives of the overall project, it may be necessary to split the sample population into two or more subsets, such as open water, freezeup, solid ice cover, and breakup. Separate frequency distributions could be derived for any population subset, but in most cases a single, annual flood-frequency distribution is desired. As described by Morris (1982), this could be derived in two ways. If the annual frequency curve is derived directly from annual peak data that have not been segregated, it is called a *mixed-population frequency curve*. If the annual frequency curve is derived from two or more frequency curves developed from separate populations, it is called a *combined-population frequency curve*. The combined-population approach should be used when frequency curves derived from mixed populations exhibit sudden breaks in curvature (Morris 1982). These sudden breaks are often caused by several large events that depart from the trend of the remainder of the data and may arise from hurricane events in a normal rainfall series, rainfall events in a snowmelt dominated series, or, in the present case, ice-influenced flooding. Details on mixed-population analysis can be found in Morris (1982) and in a publication of the Interagency Advisory Committee on Water Data (U.S. Geological Survey 1982).

a. Mixed-population frequency analysis. When determining the stage-frequency distribution for a mixed population, one must first tabulate the annual peak stage for each year of record. These annual peaks are then ranked in descending order of severity, and the exceedance probability, in terms of plotting position, is determined for each event. There are several plotting-position equations that can be used, perhaps the most common of which is the Weibull equation:

$$P = m/N + 1 \tag{5-1}$$

where

P = exceedance probability corresponding to the event of rank m

m = rank of the event

N = total number of events.

(1) The Weibull equation was developed so that the exceedance probability associated with the highest ranked event would be correct, on the average. Another commonly used

plotting-position equation, which is an approximation of the Beard, or median, plotting-position equation (Morris 1982), is

$$P = m - 0.3/N + 0.4. \tag{5-2}$$

The median plotting-position equation was developed so that the exceedance probability associated with the largest event would have an equal chance of being too high or too low.

(2) Once the plotting positions have been determined, the exceedance probability and stage coordinates are plotted on the appropriate probability paper. An analytical frequency curve may then be calculated using a selected probability distribution. The Interagency Advisory Committee on Water Data (U.S. Geological Survey 1982) recommends that the log-Pearson type III distribution, with a weighted skew coefficient, be used to model annual peak discharges. However, the interagency committee's conclusions and generalized skew coefficient map were based on annual peak data that were not segregated according to causal factors. Morris (1982) suggests that, unless the annual series in a number of stations clearly contain non-zero skew coefficients, one should use the log-normal distribution. Further, it is not clear what form of distribution ice-affected stages should follow. In view of these uncertainties, it is suggested here that the log-normal distribution be used, owing to its simplicity, with all stages referenced to the zero-discharge stage. However, since ice-affected stages are primarily governed by the ice regime and its interaction with channel geometry, extrapolation beyond the range of observed data is risky. Because discharges are normally low for ice events, the frequency curve can become highly nonlinear as flow enters the floodplain. Further, as discharge increases, a point will be reached where no stationary ice accumulation is possible and a discontinuous distribution of stages can occur as the dominant factor governing stage reverts from ice processes to water discharge. As such, it is normally sufficient to graphically fit a curve through data plotted on log-normal paper. The upper limits on ice-affected stages are discussed later.

b. Combined-population frequency analysis. For a combined-population frequency analysis, the annual peaks for each subpopulation must be tabulated similarly to the way described above for the mixed-population case.

(1) The first step is to identify the significant causal factors and determine a method for separating events into subsets. One option is to separate data populations by season (such as open water, ice covered, freezeup jamming, and breakup jamming). Such seasons must be based on different hydrometeorological conditions and not be arbitrary periods, such as calendar months. However, if the data are separated into too many subsets, one or more of the subsets may contain a few large events and many small ones (Morris 1982). This causes the frequency curves of these subsets to be unreasonably steep, and a combined annual curve will predict unreasonably high magnitudes for extreme events.

(2) The procedure for combining multiple frequency curves developed from independent annual series can be expressed in general form as

$$P_c = 1 - (1 - P_1)\,(1 - P_2)\,...\,(1 - P_n) \tag{5-3}$$

$$= 1 - \pi_{i=1}^{n} (1 - P_i) \tag{5-4}$$

where

P_c = exceedance probability of the combined-population frequency curve for the selected discharge

$P_1, P_2, \ldots P_n$ = exceedance probabilities associated with selected discharge from curve numbers 1, 2,through *n*

n = number of frequency curves to be combined.

If only two curves are combined, this reduces to

$$P_c = P_1 + P_2 - (P_1)(P_2) \tag{5-5}$$

These equations assume that each of the frequency curves used to develop a combined curve is independent, a valid assumption when combining open-water and ice-influenced events.

5-5. Approaches for Developing Ranked Data Tabulations

The discussion above has assumed that annual peak data were available for each population to be analyzed. In this paragraph, we consider how the ranked data tabulations can be developed, given the variability in data quantity and quality mentioned previously. There are two general approaches. The first is a direct analysis of historical data. The second is an indirect analysis based on data synthesized from estimates of discharge and an understanding of ice jam mechanics. The latter method is significantly more difficult than it is for the case of open-water flooding, but at the same time is typically more necessary because of the likelihood that few or no historical ice-affected data will be available. Further, it is the only feasible approach if the ice regime has changed or will be changed as a result of project construction that makes historical data obsolete. Frequently, the best approach is to use a combination of the direct and indirect methodologies.

a. Direct approach. If reliable hydrometric data are available at the site, and the desired product is a combined-frequency analysis of the open-water and ice-covered flows, the analysis is relatively straightforward. The maximum open-water and ice-covered stages should be tabulated for each water year. The two event types are independent because they have different causes (water quantity versus ice processes) and are not mutually exclusive (an open-water flood can occur in the same year as an ice-affected flood). Each of these populations can then be analyzed using standard techniques, as discussed earlier. If the desired subpopulation is ice jam events, however, a variation in technique is required, since ice jams do not typically form every year at a given site.

(1) Morris (1982) discusses several options for the somewhat comparable case of analyzing hurricane events. One applicable approach is to redefine the number of events in the plotting-position formula as being equal to the total number of years of record analyzed. This tacitly as-

sumes that the record is continuous and that all events in the subpopulation have been identified. Morris also suggests that the frequency curve be developed by drawing a best-fit line by eye, rather than using regression equations, so that outliers do not unduly affect the derived line. Because of the small sample size typical of this type of analysis, there can be significant uncertainty in the accuracy of the resultant frequency curve.

(2) Very often, there is no long-term, reliable gaging station at the project site, and it becomes necessary to combine information from several sources of varying accuracy and reliability. There may also be years in which no data were recorded, but it is not clear whether there was no event or whether it simply was not recorded. This inhomogeneity of data can reduce the reliability of the resulting frequency curve. If the record is clearly incomplete, consideration should be given to employing the indirect method of analysis, with checks against the available historical data, as discussed later.

(3) When analyzing a data set with multiple data sources, it is necessary to determine which one is the most reliable, when more than one data source records an event, and also to determine the maximum stage that may have occurred in years when there were no reports on ice-affected stages. While there is no standard technique for this integration of data sources, a reasonable methodology has been given by Gerard and Karpuk (1979), as outlined briefly below.

(a) *Perception stage*. One must determine the minimum stage (or *perception stage*) that would be recorded by various data sources. This perception stage is defined as the minimum stage required for a given source to perceive and record an event. For example, for a gaging station, the perception stage would be the minimum stage it was capable of recording, while for a local resident, it would be that stage below which the event would have gone unnoticed or unremembered. Newspaper accounts would require an event of interest to its readers, and photographs would require an event significant enough for a person to want a permanent record (unless the event was captured incidental to a different subject). The significance of the perception stage is that, if a source was in a position to observe events during a given year, but didn't report any, then one can presume that the maximum stage during that year was less than the perception stage of the source. This simple constraint provides an objective means of merging data from several sources and of increasing the record length beyond the recorded number of events.

(b) *Environmental indicators of stage*. Environmental indicators, such as structural markings and high-water marks, can be used, providing one knows when they happened. The same is typically true for other environmental evidence, except that it is sometimes possible to date old tree scar data by analyzing tree rings. This has been used with success in prior studies to document high-water levels that were several decades old. In either case, analysis can be complex, since a stable object would have to have been located at a proper location and elevation to allow recording, and the markings would have had to be high enough and long lasting enough to persist through subsequent floods.

(c) *Record length*. In this method, the record length will vary with stage. Record length for an individual peak is equal to the number of years in which any source with a perception stage lower than that peak was present on the river. For example, assume that the hypothetical data in Table 5-1 came from three different sources, such as 1) the memory of a local resident, 2) an early water-level gage, and 3) a more recent water-level gage. Perception stages of 2.7, 0.9, and 0 meters (9, 3, and 0 feet), relative to a zero-flow stage, have been assigned to these sources. Further, assume that the local resident was present throughout the period of record, but that the first water-level gage operated only in years 7–14 and the final gage operated in years 15–20, except when it malfunctioned during year 19. Under this scenario, if the third source recorded a stage of 0.88 meters (2.89 feet), the record length associated with that event would correspond to only the 5 years for which that source reported data. If the first data source had reported a stage of 3.05 meters (10 feet) during the first 6 years, the record length for that event would equal the total number of years that any of the three sources were active, since any of the three sources would have recorded the event had they been present.

(d) *Stage ranking.* This method also requires a modified technique for determining the rank of peak stages. When the data are tabulated for such a scenario, the rank for an individual peak is determined by ranking all peaks having a perception stage less than or equal to the value of the peak. Thus, peaks above 2.7 meters (9 feet) would be ranked in terms of the entire data set, but peaks between 0.9 and 2.7 meters (3 and 9 feet) would only be ranked with those events from the second and third sources, since an event of that magnitude may have occurred during the years when only the first source was active and gone unnoticed. As noted in Table 5-2, this can result in two or more peaks being assigned equal rank, but their record length will differ.

(4) Plotting positions can now be calculated using standard formulations and the redefined values of record lengths and rank, and the stage-frequency distribution determined as before. Although this method allows a logical means of combining inhomogeneous data sets, the discontinuities in record lengths and rank can persist as discontinuities in plotting positions. Although there are a number of available plotting-position formulas (the Weibull simulation was used in Table 5-2), they all basically divide rank by record length without any other reference to the associated stage. Thus, it is possible for an event of record length 20 and rank 7 to have the same calculated plotting position as an event of record length 14 and rank 5. For example, if the missing record from year 19 had an actual stage of less than 2.48 meters (8.15 feet), then the record length for the event in year 16 would have been 14 and the plotting position would have been 0.333, equal to that for the event in year 14. By similar reasoning, it is possible to calculate a given stage as being more probable than a lesser stage, which is clearly not realistic. If such an overlap of data groups occurs, it is generally slight, and is best treated as data scatter with the final frequency-distribution curve smoothed by eye.

Table 5-1
Example Historic Data Set

Year	Data Source*	Perception Stage m	Perception Stage ft	Stage m	Stage ft	Record Length
1	1	2.7	9	3.20	10.5	20
2	1	2.7	9	——	—	—
3	1	2.7	9	——	—	—
4	1	2.7	9	——	—	—
5	1	2.7	9	3.66	12.0	20
6	1	2.7	9	3.05	10.0	20
7	1,2	0.9	3	——	—	—
8	1,2	0.9	3	4.59	15.05	20
9	1,2	0.9	3	0.96	3.15	13
10	1,2	0.9	3	1.18	3.88	13
11	1,2	0.9	3	1.00	3.28	13
12	1,2	0.9	3	1.19	3.89	13
13	1,2	0.9	3	——	—	—
14	1,2	0.9	3	2.88	9.46	20
15	1,3	0.0	0	6.58	21.59	20
16	1,3	0.0	0	2.48	8.15	13
17	1,3	0.0	0	0.88	2.89	5
18	1,3	0.0	0	5.22	17.12	20
19	1	2.7	9	——	—	—
20	1,3	0.0	0	1.97	6.45	13

* Key:
1. Memory of a local resident
2. Early water-level gage
3. Recent water-level gage

b. Indirect method. The indirect method of stage-frequency analysis uses stage data synthesized from estimates of discharge frequency and knowledge of ice processes. While data synthesis is more difficult than in the open-water case, it is also more necessary because of the general lack of appropriately sited gaging stations or other sources of historical data. Further, it is the only feasible approach if the ice regime has changed or will be changed, making historical data obsolete. Major obstacles to be overcome include estimating the appropriate ice conditions and assessing the frequency of ice jamming at a particular site.

Table 5-2
Example Data Tabulation

Year	*Stage, m (ft)*		*Perception Stage, m (ft)*		*Record Length*	*Rank*	*Plotting Position*
15	6.58	(21.59)	0.0	(0)	20	1	0.048
18	5.22	(17.12)	0.0	(0)	20	2	0.095
8	4.59	(15.05)	0.9	(3)	20	3	0.143
5	3.66	(12.0)	2.7	(9)	20	4	0.190
1	3.20	(10.5)	2.7	(9)	20	5	0.238
6	3.05	(10.0)	2.7	(9)	20	6	0.286
14	2.88	(9.46)	0.9	(3)	20	7	0.333
16	2.48	(8.15)	0.0	(0)	13	5	0.357
20	1.97	(6.45)	0.0	(0)	13	6	0.429
12	1.19	(3.89)	0.9	(3)	13	7	0.500
10	1.18	(3.88)	0.9	(3)	13	8	0.571
11	1.00	(3.28)	0.9	(3)	13	9	0.643
9	0.96	(3.15)	0.9	(3)	13	10	0.714
17	0.88	(2.89)	0.0	(0)	5	5	0.833

(1)As in the open-water case, discharge and meteorological data may be used to generate probable ice-related events for each year of record. The period stage-frequency distribution is then developed using the appropriate ice-cover-period or ice-jam-period discharge frequency, available ice data, an analysis of probable ice-related water levels (i.e., HEC-RAS, see Chapter 4), and some estimate of jam frequency.

(2) The first step is a year-by-year analysis of flow records to determine the maximum annual discharge for each desired subpopulation. Ideally, these values would reflect the instantaneous peak flows, since they are the ones that determine the severity of ice effects. On the other hand, a careful review of the records is required to ensure that the flows are from the ice-jam period, and not an open-water peak following the final breakup ice run. If necessary, these data may be transposed from gage data elsewhere on the river, transposed from other rivers in the vicinity, or estimated using a precipitation-runoff model. Next, representative ice conditions must be estimated for the range of expected breakup events. Such information might include estimates of ice thickness, ice-cover or ice-jam roughness, position of the ice jam's toe and head, and the upstream length of river contributing ice to a jam.

(3) Lacking field data, it is very difficult to predict where, and with what frequency, jams will form along a river, and analysis is often limited to estimating upper and lower limits of probable stages. If a jam is known (or assumed) to form at a given location, it is possible to estimate the maximum resulting flood levels. It can be shown that, for a given scenario of water discharge and ice conditions, the maximum water levels will occur within the equilibrium portion of the jam described earlier. Since ice and flow conditions are relatively uniform within the

equilibrium reach, it is a fairly simple matter to estimate the water levels in this portion of the jam. Depending on where a jam forms, and whether there is a sufficient upstream ice discharge to form a jam long enough to develop an equilibrium reach, actual water levels may be less and the estimate will be conservative.

(4) Thus, if no site information is available, the range of possible ice conditions might be assumed to include the limiting conditions of a solid cover of sheet ice and a fully developed equilibrium ice jam. The solid-ice-cover case would represent the minimum ice-affected stage, while the equilibrium-ice-jam case would represent the maximum stage possible for a given discharge. If, for example, we were to assume that the problem at hand is an analysis of flooding attributable to breakup jams in the spring, and that little or no information exists for the ice regime, a suggested procedure is as follows.

(a) *List peak flows.* Develop a table of peak flows for the period of breakup, as discussed above. These flows should be estimates of the instantaneous peak flows, since they are the ones that govern the maximum severity of the event.

(b) *Define range of flows.* From this tabulation of flows, select a range of flows from discharges too low to cause breakup of the ice cover to discharges where all ice would move downstream without jamming. These estimates might be based on personal observations, observations by local residents, notes on nearby gaging records, sharp breaks in the trend of continuous stage measurements, or other sources of information. These estimates might also vary through the winter because of variations in ice strength. If such estimates are not possible, select a range of flows representative of all historical breakup period flows.

(c) *Calculate stages*. For a number of discharges covering the range of flows defined in the preceding paragraph, calculate the stages associated with both the solid-ice-cover and equilibrium-ice-jam cases using either a numerical model such as HEC-RAS or manually using a procedure such as the one outlined in Beltaos (1983).

(d) *Develop rating curve*. Using the above information, develop a stage-discharge rating curve for both the lower bound of a solid ice cover and the upper bound of an equilibrium ice jam.

(e) *Rank stages.* From the tabulation of historical discharges and the stage-discharge rating curve, develop a ranked tabulation of estimated historical stage data. As in the direct method, a stage-probability distribution may now be developed for the upper and lower bound cases by assigning plotting positions and plotting on log-normal paper.

(5) The task of developing a compromise distribution from these upper and lower bound distributions remains, and one should consider limiting conditions on the stage-discharge relations. The first limit is the discharge (or stage) at which the ice cover is expected to break up, as described above in paragraph 5-5*b*(4)(b). If this lower limit can be estimated, then it can be assumed that the solid-ice-cover curve is appropriate for all discharges below that level.

(6) If a large floodplain exists at the site, it may provide an upper limit on ice-induced stages. Since ice-related discharges are typically low relative to open-water events, once the stage is high enough to allow water to enter the floodplain, the slope of the stage-discharge curve should flatten considerably. Further, with continued increases in discharge, a point may be reached at which no stable ice jam is possible—all ice will simply be transported downstream. The development of an ice jam can also be limited by the volume of ice available for accumulation. If there is some physical limit to the upstream river length contributing ice (e.g., the presence of an upstream dam), there may be an insufficient ice supply to develop an equilibrium jam at the project site. Thus, extrapolation of ice-induced stages to more extreme events is generally not reliable, particularly if such limiting factors are not considered.

(7) Beyond imposing such limits, developing a compromise distribution between the ice-cover and ice-jam distributions is largely a matter of engineering judgment. If the results are to be used for project design, it might be desirable to conservatively assume that ice jams always form and employ that distribution. However, for the determination of average annual damages, even a few reliable historical data could be of immense help in the interpretation of the two analytically derived distributions. By comparing a historical observation with the analytical estimates for a comparable event (and using judgment), a compromise best-estimate of the stage for an event of that magnitude can be developed.

(8) If no such data are available, some idea of the frequency of significant ice jamming in the vicinity of the site can be helpful (e.g., do jams occur every other year, or in about 3 out of 10 years?). Although not related to specific years, this general information can be used in the development of a compromise curve (Figure 5-2) by employing the methodology of Gerard and Calkins (1984) as follows:

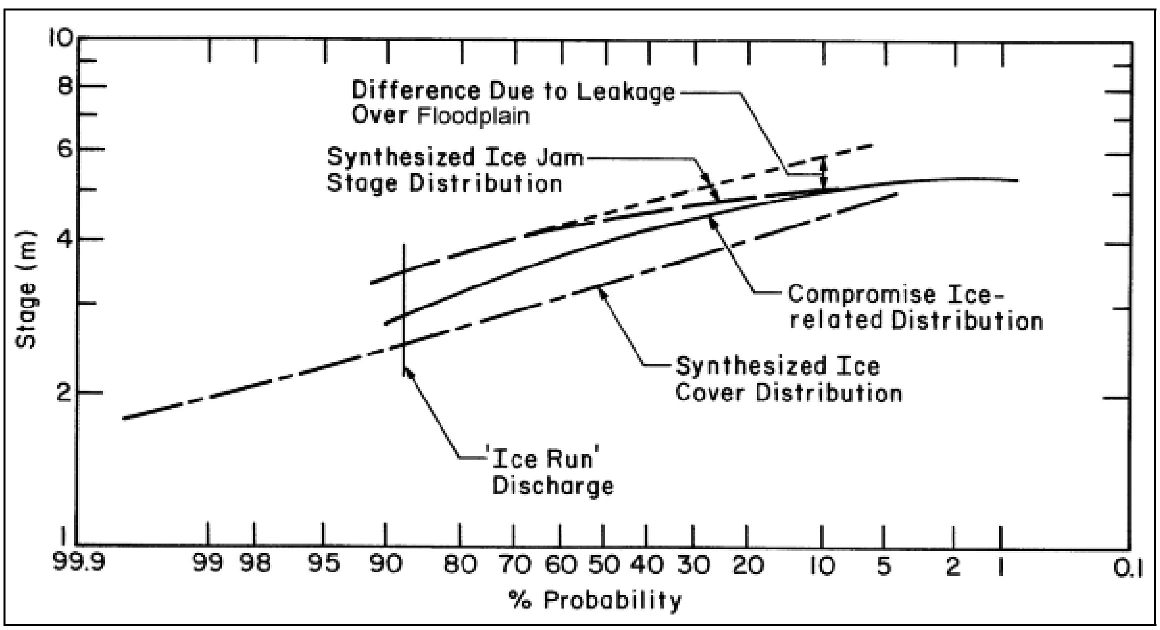

Figure 5-2. Development of a compromise distribution

(a) *Probability of stage exceedance because of jamming.* If the probability that a significant ice jam may form in a given year is $P(J) = m/N$ (m being the estimated number of jam events in N years), then the probability $P(\$_J)$ of a given stage being equaled or exceeded in a given year is given by

$$P(\$_J) = P(J) \cdot P(\$ \mid J) \tag{5-6}$$

where $P(\$ \mid J)$ is the probability of the stage being exceeded if an ice jam forms (i.e., a conditional probability). $P(\$ \mid J)$ corresponds to the probability coordinate of the upper bound for a given stage.

(b) *Probability of stage exceedance because of a solid ice cover.* The probability $P(\$_C)$ of the stage being exceeded when a solid ice cover exists is likewise given by

$$P(\$_C) = P(C) \cdot P(\$ \mid C) \tag{5-7}$$

where $P(C) = 1 - P(J)$ is the probability of a significant ice jam not occurring (and therefore the peak stage being associated with a solid ice cover), and $P(\$ \mid C)$ is the conditional probability of the stage being exceeded if a significant ice jam does not form (the lower bound).

(c) *Probability of stage exceedance because of ice in general.* As the ice cover and ice jam situations are mutually exclusive, the probability $P(\$)$ of a stage being equaled or exceeded is given by

$$P(\$) = P(\$_J) + P(\$_C). \tag{5-8}$$

Thus, if a jam forms in about 3 out of every 10 years, $P(J) = 0.3$ and

$$P(C) = 1 - P(J) = 1 - 0.3 = 0.7.$$

For a given stage, $P(\$ \mid J)$ and $P(\$ \mid C)$ are determined from the upper and lower bound frequency curves, respectively. Equation 5-8 can then be used to calculate the compromise probability $P(\$)$ of a given stage being exceeded.

(9) Repeating this procedure for the range of breakup flows allows the development of a compromise frequency curve between the upper and lower bounds. However, at the lower end it should merge with the solid-ice-cover case at a point where the discharge would be too low to cause ice-cover breakup, and the upper end must be reconciled with the limits imposed by floodplain flow or a finite ice supply as discussed earlier. It must be emphasized that extrapolation of ice-induced stages to extreme events is generally not reliable, particularly if such limiting factors have not been considered.

5-6. Summary

This chapter has reviewed methodologies for the analysis of ice-related flood frequency. Since ice-induced flooding is dominated by ice processes, rather than water quantity, we have emphasized the need for a stage-related, rather than discharge-related, analysis. Further, since detailed data for ice-affected events are typically unavailable, and the site-specific nature of ice-related flooding generally precludes transposing data from other sites, methodologies are outlined for performing the analysis based on limited historical data and data synthesized from discharge records and estimates of ice conditions.

5-7. References

a. Required publications.

None.

b. Related publications.

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